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DISTANCE COPRIME SYMMETRIC N-GRAPHS

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Abstract

An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function. In this paper, we introduced a new notion distance coprime symmetric n-sigraph of a symmetric n-sigraph and its properties are obtained. Also, we obtained the structural characterization of distance coprime symmetric n-signed graphs.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [1]. We consider only finite, simple graphs free from self-loops.

Key Words : *Symmetric n-sigraphs, Symmetric n-marked graphs, Balance, Switching, Distance Coprime symmetric n-sigraphs, Complementation.*

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Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [7], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [3]):

Definition : Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [7].

Theorem 1.1 (**E. Sampathkumar et al. [7]**) : An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

In [7], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [2], [4-6], [9-19]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [7]).

Theorem 1.2 (E. Sampathkumar et al. [7]) : Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S defined as follows: each vertex $v \in V$, $\mu(v)$ is the product of the n-tuples on the edges incident at v. *Complement* of S is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Theorem 1.1.

2. Distance Coprime n-Sigraph of an n-Sigraph

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. The shortest path P in G is said to be distance coprime path, if $gcd(l(P), q) = 1$, where $l(P)$ denotes the length path P.

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. The distance coprime graph $\mathcal{DCP}(G)$ of $G = (V, E)$ is a graph with $V(\mathcal{DCP}(G)) = V(G)$ and any two vertices u and v in $\mathcal{DCP}(G)$ are joined by an edge if there exists a distance coprime path between them in G. This concept were introduced by Suganya and Nagarajan [20].

Motivated by the existing definition of complement of an n-sigraph, we extend the notion of distance coprime graphs to n-sigraphs as follows: The *distance coprime n-sigraph* $\mathcal{DCP}(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $\mathcal{DCP}(G)$ and the n-tuple of any edge uv is $\mathcal{DCP}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Further, an n-sigraph $S_n = (G, \sigma)$ is called distance coprime n-sigraph, if $S_n \cong \mathcal{DCP}(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion $\mathcal{DCP}(S_n)$ as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be distance coprime n-sigraphs.

Theorem 2.1 : For any n-sigraph $S_n = (G, \sigma)$, its distance coprime n-sigraph $\mathcal{DCP}(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $\mathcal{DCP}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $\mathcal{DCP}(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated distance coprime n-sigraph $\mathcal{DCP}(S_n)$ of S_n is defined as follows:

$$(\mathcal{DCP})^0(S_n) = S_n,\ (\mathcal{DCP})^k(S_n) = \mathcal{DCP}((\mathcal{DCP})^{k-1}(S_n)).$$

Corollary 2.2 : For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(\mathcal{DCP})^k(S_n)$ is i-balanced.

The following result characterize n-sigraphs which are distance coprime n-sigraphs.

Theorem 2.3 : An n-sigraph $S_n = (G, \sigma)$ is a distance coprime n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a distance coprime graph.

Proof : Suppose that S_n is i-balanced and G is a $\mathcal{DCP}(G)$. Then there exists a graph H such that $\mathcal{DCP}(H) \cong G$. Since S_n is i-balanced, by Theorem 1.1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $\mathcal{DCP}(S'_n) \cong S_n$. Hence S_n is a distance coprime n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a distance coprime n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $\mathcal{DCP}(S'_n) \cong S_n$. Hence G is the $\mathcal{DCP}(G)$ of H and by Theorem 2.1, S_n is i-balanced. □

In [20], the authors characterizes the graphs such that G and $\mathcal{DCP}(G)$ are isomorphic.

Theorem 2.4 : Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$, where q is a composite number. Then G and $\mathcal{DCP}(G)$ are isomorphic if and only if the diameter of G is less than or equal to $c_2 - 1$, where c_2 is the second coprime of q.

In view of the above, we have the following result:

Theorem 2.5 : For any n-sigraph $S_n = (G, \sigma)$ with $|V| = p$ and $|E| = q$, where q is a composite number. Then S_n and $\mathcal{DCP}(S_n)$ are cycle isomorphic if and only if S_n is

i-balanced and the diameter of G is less than or equal to $c_2 - 1$, where c_2 is the second coprime of q.

Proof : Suppose $\mathcal{DCP}(S_n) \sim S_n$. This implies, $\mathcal{DCP}(G) \cong G$ and hence by Theorem 2.4, we see that the diameter of G is less than or equal to $c_2 - 1$, where c_2 is the second coprime of q. Now, if S is any n-sigraph with diameter of G is less than or equal to $c_2 - 1$, where c_2 is the second coprime of q. Then $\mathcal{DCP}(S_n)$ is i-balanced and hence if S_n is i-unbalanced and its distance coprime n-sigraph $\mathcal{DCP}(S_n)$ being i-balanced can not be switching equivalent to S_n in accordance with Theorem 1.2. Therefore, S_n must be i-balanced.

Conversely, suppose that S balanced signed graph with the underlying graph G satisfies the conditions of Theorem 2.4. Since $\mathcal{DCP}(S)$ is balanced as per Theorem 2.1, the result follows from Theorem 1.2 again. □

3. Complementation

In this section, we investigate the notion of complementation of a graph whose edges have signs (a *sigraph*) in the more general context of graphs with multiple signs on their edges. We look at two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2, ..., a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the *m-complement* of an n-sigraph $S_n = (G, \sigma)$, written (S_n^m), is the same graph but with each edge label $a = (a_1, a_2, ..., a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $\mathcal{DCP}(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $\mathcal{DCP}(S_n)$ is i-balanced, where for any $m \in H_n$.

Theorem 3.1 : Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $\mathcal{DCP}(G)$ is bipartite then $(\mathcal{DCP}(S_n))^m$ is i-balanced.

Proof : Since, by Theorem 2.1, $\mathcal{DCP}(S_n)$ is i-balanced, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $\mathcal{DCP}(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $\mathcal{DCP}(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $\mathcal{DCP}(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m, \in H_n$. Hence $(\mathcal{DCP}(S_n))^t$ is i-balanced. □

4. Conclusion

We have introduced a new notion for n-signed graphs called distance coprime n-sigraph of an n-signed graph. We have proved some results and presented the structural characterization of distance coprime n-signed graph. There is no structural characterization of distance coprime graph, but we have obtained the structural characterization of distance coprime n-signed graph.

References

[1] Harary F., Graph Theory, Addison-Wesley Publishing Co., (1969)0.

[2] Lokesha V., Reddy P.S.K. and Vijay S., The triangular line n-sigraph of a symmetric n-sigraph, Advn. Stud. Contemp. Math., 19(1) (2009), 123-129.

[3] Rangarajan R. and Reddy P.S.K., Notions of balance in symmetric n-sigraphs, Proceedings of the Jangjeon Math. Soc., 11(2) (2008), 145-151.

[4] RangarajanR., Reddy P.S.K. and Subramanya M. S., Switching Equivalence in Symmetric n-Sigraphs, Adv. Stud. Comtemp. Math., 18(1) (2009), 79-85.

[5] Rangarajan R., Reddy P.S.K. and Soner N. D., Switching equivalence in symmetric n-sigraphs-II, J. Orissa Math. Sco., 28 (1 & 2) (2009), 1-12.

[6] Rangarajan R., Reddy P.S.K. and Soner N. D., m^{th} Power Symmetric n-Sigraphs, Italian Journal of Pure & Applied 985099211 Mathematics, 29(2012), 87-92.

[7] Sampathkumar E., Reddy P.S.K. and Subramanya M. S., Jump symmetric n-sigraph, Proceedings of the Jangjeon Math. Soc., 11(1) (2008), 89-95.

[8] Sampathkumar E., Reddy P.S.K., and Subramanya M. S., The Line n-sigraph of a symmetric n-sigraph, Southeast Asian Bull. Math., 34(5) (2010), 953-958.

[9] Reddy P.S.K. and Prashanth B., Switching equivalence in symmetric n-sigraphs-I, Advances and Applications in Discrete Mathematics, 4(1) (2009), 25-32.

[10] Reddy P.S.K., Vijay S. and Prashanth B., The edge C_4 n-sigraph of a symmetric n-sigraph, Int. Journal of Math. Sci. & Engg. Appls., 3(2) (2009), 21-27.

[11] Reddy P.S.K., Lokesha V. and Gurunath Rao Vaidya, The Line n-sigraph of a symmetric n-sigraph-II, Proceedings of the Jangjeon Math. Soc., 13(3) (2010), 305-312.

[12] Reddy P.S.K., Lokesha V. and Gurunath Rao Vaidya, The Line n-sigraph of a symmetric n-sigraph-III, Int. J. Open Problems in Computer Science and Mathematics, 3(5) (2010), 172-178.

[13] Reddy P.S.K., Lokesha V. and Gurunath Rao Vaidya, Switching equivalence in symmetric n-sigraphs-III, Int. Journal of Math. Sci. & Engg. Appls., 5(1) (2011), 95-101.

[14] Reddy P.S.K., Prashanth B. and Kavita. S. Permi, A Note on Switching in Symmetric n-Sigraphs, Notes on Number Theory and Discrete Mathematics, 17(3) (2011), 22-25.

[15] Reddy P.S.K., Geetha M. C. and Rajanna K. R., Switching Equivalence in Symmetric n-Sigraphs-IV, Scientia Magna, 7(3) (2011), 34-38.

[16] Reddy P.S.K., Nagaraja K. M. and Geetha M. C., The Line n-sigraph of a symmetric n-sigraph-IV, International J. Math. Combin., 1 (2012), 106-112.

[17] Reddy P.S.K., Geetha M. C. and Rajanna K. R., Switching equivalence in symmetric n-sigraphs-V, International J. Math. Combin., 3 (2012), 58-63.

[18] Reddy P.S.K., Nagaraja K. M. and Geetha M. C., The Line n-sigraph of a symmetric n-sigraph-V, Kyungpook Mathematical Journal, 54(1) (2014), 95-101.

[19] Reddy P.S.K., Rajendra R. and Geetha M. C., Boundary n-Signed Graphs, Int. Journal of Math. Sci. & Engg. Appls., 10(2) (2016), 161-168.

[20] Suganya R. and Nagarajan K., Distance coprime graphs, International J. of Pure & Engg. Mathematics, 4(1) (2016), 19-30.